

Floating Point Routines for the 6502

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Editor's Note: Although these routines are for the 6502, it would appear that one could generate equivalent routines for most of the "traditional" microprocessors, relatively easily, by following the flow of the algorithms given in the excellent comments included in the program listing. This is particularly true of the transcendental functions which were directly modeled after well-known and proven algorithms, and for which, the comments are relatively machine-independent.

These floating point routines allow 6502 users to perform most of the more popular and desired floating point and transcendental functions, namely:

- Natural Log - LOG
- Common Log - LOG10
- Exponential - EXP
- Floating Add - FADD
- Floating Subtract - FSUB
- Floating Multiply - FMUL
- Floating Divide - FDIV
- Convert Floating to Fixed - FIX
- Convert Fixed to Floating - FLOAT

They presume a four-byte floating point operand consisting of a one-byte exponent ranging from -218 through +127, and a 24-bit two's complement mantissa between 1.0 and 2.0.

The floating point routines were done by Steve Wozniak, one of the principals in Apple Computer Company. The transcendental functions were patterned after those offered by Hewlett-Packard for their HP2100 minicomputer (with some modifications), and were done by Roy Rankin, a Ph.D. student at Stanford University.

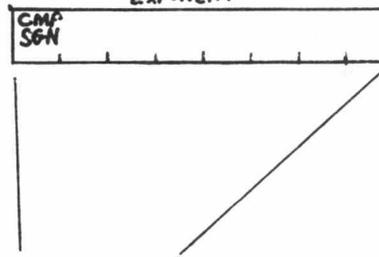
There are three error traps; two for overflow, and one for prohibited logarithm argument. ERROR (1D06) is the error exit used in event of a non-positive log argument. OVFLW (1E3B) is the error exit for overflow occurring during calculation of e to some power. OVFL (1FE4) is the error exit for overflow in all of the floating point routines. There is no trap for underflow; in such cases, the result is set to 0.0.

All routines are called and exited in a uniform manner: The argument(s) are placed in the specified floating point storage locations (for specifics, see documentation preceding each routine in the listing), then a JSR is used to enter the desired routine. Upon normal completion, the called routine is exited via a subroutine return instruction (RTS).

Note: The preceding documentation was written by the Editor, based on phone conversations with Roy and studying the listing. There is a high probability that it is correct. However, since it was not written nor reviewed by the authors of these routines, the preceding documentation may contain errors in concept or in detail.

- JCW, Jr.

0 is -
1 is +
EXPONENT

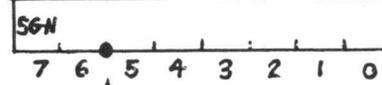


In the Exponent:
00 Represents -128
:
7F Represents -1
80 Represents 0
81 Represents +1
:
FF Represents +127

EXPONENT TWO'S COMPLEMENT MANTISSA

n $n+1$ $n+2$ $n+3$

M.SIG. BYTE OF MANTISSA



PRESUMED DECIMAL POINT

0 is +
1 is -

* JULY 5, 1976
* BASIC FLOATING POINT ROUTINES
* FOR 6502 MICROPROCESSOR
* BY R. RANKIN AND S. WOZNIAK

* CONSISTING OF:
* NATURAL LOG
* COMMON LOG
* EXPONENTIAL (E**X)
* FLOAT FIX
* FADD FSUB
* FMUL FDIV

* FLOATING POINT REPRESENTATION (4-BYTES)
* EXPONENT BYTE 1
* MANTISSA BYTES 2-4

* MANTISSA: TWO'S COMPLIMENT REPRESENTATION WITH SIGN IN
* MSB OF HIGH-ORDER BYTE. MANTISSA IS NORMALIZED WITH AN
* ASSUMED DECIMAL POINT BETWEEN BITS 5 AND 6 OF THE HIGH-ORDER
* BYTE. THUS THE MANTISSA IS IN THE RANGE 1. TO 2. EXCEPT
* WHEN THE NUMBER IS LESS THAN 2**(-128).

* EXPONENT: THE EXPONENT REPRESENTS POWERS OF TWO. THE
* REPRESENTATION IS 2'S COMPLIMENT EXCEPT THAT THE SIGN
* BIT (BIT 7) IS COMPLIMENTED. THIS ALLOWS DIRECT COMPARISON
* OF EXPONENTS FOR SIZE SINCE THEY ARE STORED IN INCREASING
* NUMERICAL SEQUENCE RANGING FROM \$00 (-128) TO \$FF (+127)
* (\$ MEANS NUMBER IS HEXADECIMAL).

* REPRESENTATION OF DECIMAL NUMBERS: THE PRESENT FLOATING
* POINT REPRESENTATION ALLOWS DECIMAL NUMBERS IN THE APPROXIMATE
* RANGE OF 10**(-38) THROUGH 10**(38) WITH 6 TO 7 SIGNIFICANT
* DIGITS.

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0003          ORG 3          SET BASE PAGE ADDRESSES
0003 EA      SIGN  NOP
0004 EA      X2   NOP
0005 00 00 00 M2   BSS 3    MANTISSA 2
0008 EA      X1   NOP
0009 00 00 00 M1   BSS 3    MANTISSA 1
000C          E    BSS 4    SCRATCH
0010          Z    BSS 4
0014          T    BSS 4
    
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0018 SEXP BSS 4
001C INT BSS 1
*
1D00 * ORG $1D00 STARTING LOCATION FOR LOG
*
* NATURAL LOG OF MANT/EXP1 WITH RESULT IN MANT/EXP1
*
1D00 A5 09 LOG LDA M1
1D02 F8 02 BEO ERROR
1D04 10 01 BPL CONT IF ARG>0 OK
1D06 00 ERROR BRK ERROR ARG<=0
*
1D07 20 1C IE CONT JSR SWAP MOVE ARG TO EXP/MANT2 < A2.00 LDX # 0
1D0A A5 04 LDA X2 HOLD EXPONENT
1D0C A0 00 LDY =#00
1D0E 04 04 STY X2 SET EXPONENT 2 TO 0 (#00)
1D10 49 00 EOR =#00 COMPLIMENT SIGN BIT OF ORIGINAL EXPONENT
1D12 05 0A STA M1+1 SET EXPONENT INTO MANTISSA 1 FOR FLOAT
1D14 A9 00 LDA =0 BPL *+3
1D16 05 09 STA M1 CLEAR MSB OF MANTISSA 1 DEX < 86 09 STX M1
1D18 20 2C IF JSR FLOAT CONVERT TO FLOATING POINT
1D1B A2 03 LDX =3
1D1D 05 04 SEXP1 LDA X2.X
1D1F 95 10 STA Z.X COPY MANTISSA TO Z
1D21 05 00 LDA X1.X
1D23 95 10 STA SEXP.X
1D25 0D 11 ID LDA R22.X SAVE EXPONENT IN SEXP
1D28 95 00 STA X1.X LOAD EXP/MANT1 WITH SORT(2)
1D2A CA DEX
1D2B 10 F0 BPL SEXP1
1D2D 20 4A IF JSR FSUB Z-SORT(2)
1D30 A2 03 LDX =3 4 BYTE TRANSFER
1D32 05 00 SAVET LDA X1.X SAVE EXP/MANT1 AS T
1D34 95 14 STA T.X
1D36 05 10 LDA Z.X LOAD EXP/MANT1 WITH Z
1D38 95 00 STA X1.X
1D3A 0D 11 ID LDA R22.X LOAD EXP/MANT2 WITH SORT(2)
1D3D 95 04 STA X2.X
1D3F CA DEX
1D40 10 F0 BPL SAVET
1D42 20 50 IF JSR FADD Z+SORT(2)
1D45 A2 03 LDX =3 4 BYTE TRANSFER
1D47 05 14 TM2 LDA T.X
1D49 95 04 STA X2.X LOAD T INTO EXP/MANT2
1D4B CA DEX
1D4C 10 F9 BPL TM2
1D4E 20 9D IF JSR FDIV T=(Z-SORT(2))/(Z+SORT(2))
1D51 A2 03 LDX =3 4 BYTE TRANSFER
1D53 05 00 MIT LDA X1.X
1D55 95 14 STA T.X COPY EXP/MANT1 TO T AND
1D57 95 04 STA X2.X LOAD EXP/MANT2 WITH T
1D59 CA DEX
1D5A 10 F7 BPL MIT
1D5C 20 77 IF JSR FMUL T*T
1D5F 20 1C IF JSR SWAP MOVE T*T TO EXP/MANT2
1D62 A2 03 LDX =3 4 BYTE TRANSFER
1D64 0D E1 ID MIC LDA C.X
1D67 95 00 STA X1.X LOAD EXP/MANT1 WITH C
1D69 CA DEX
1D6A 10 F8 BPL MIC
1D6C 20 4A IF JSR FSUB T*T-C
1D6F A2 03 LDX =3 4 BYTE TRANSFER
1D71 0D 0D ID M2MB LDA MB.X
1D74 95 04 STA X2.X LOAD EXP/MANT2 WITH MB
1D76 CA DEX
1D77 10 F8 BPL M2MB
1D79 20 9D IF JSR FDIV MB/(T*T-C)
1D7C A2 03 LDX =3 4 BYTE TRANSFER
1D7E 0D 09 ID M2A1 LDA A1.X
1D81 95 04 STA X2.X LOAD EXP/MANT2 WITH A1
1D83 CA DEX
1D84 10 F8 BPL M2A1
1D86 20 50 IF JSR FADD MB/(T*T-C)+A1
1D89 A2 03 LDX =3 4 BYTE TRANSFER
1D8B 05 14 M2T LDA T.X
1D8D 95 04 STA X2.X LOAD EXP/MANT2 WITH T
1D8F CA DEX
1D90 10 F9 BPL M2T
1D92 20 77 IF JSR FMUL (MB/(T*T-C)+A1)*T
1D95 A2 03 LDX =3 4 BYTE TRANSFER
1D97 0D E5 ID M2MHL LDA MHLF.X
1D9A 95 04 STA X2.X LOAD EXP/MANT2 WITH MHLF (.5)
1D9C CA DEX
1D9D 10 F8 BPL M2MHL
1D9F 20 50 IF JSR FADD +.5
1DA2 A2 03 LDX =3 4 BYTE TRANSFER
1DA4 05 10 LDEXP LDA SEXP.X
1DA6 95 04 STA X2.X LOAD EXP/MANT2 WITH ORIGINAL EXPONENT
1DAB CA DEX
1DA9 10 F9 BPL LDEXP
1DAB 20 50 IF JSR FADD +EXPN
1DAE A2 03 LDX =3 4 BYTE TRANSFER
1DB0 0D D5 ID MLE2 LDA LE2.X
1DB3 95 04 STA X2.X LOAD EXP/MANT2 WITH LN(2)
1DB5 CA DEX
1DB6 10 F8 BPL MLE2
1DB8 20 77 IF JSR FMUL *LN(2)
1DBB 60 RTS RETURN RESULT IN MANT/EXP1
*
* COMMON LOG OF MANT/EXP1 RESULT IN MANT/EXP1
*
1DBC 20 00 ID LOG10 JSR LOG COMPUTE NATURAL LOG
1DBF A2 03 LDX =3

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